

Summer semester 2014

# Graduate Seminar on Numerical Simulation

Prof. Dr. Carsten Burstedde

## 1 Summary

We will discuss optimization and inverse problems with PDEs. If we define the forward problem by computing the solution of a PDE given its right hand side and system coefficients, optimization and inverse problems refer to computing right hand sides or coefficients given a certain target solution. Such problems are particularly challenging for the following reasons:

1. Even for a linear forward problem, the inverse problem can be (strongly) nonlinear.
2. The inverse problem may be ill-posed: its solution may not depend continuously on the input data, and there may be none or multiple exact solutions.
3. We may want to restrict the unknown coefficients to certain subspaces (for example, they should be non-negative if they enter the bilinear form of an elliptic operator).
4. Computationally, we solve a minimization problem where the PDE is a constraint. We will introduce Lagrange multipliers in certain function spaces. Each step of the inverse solver involves one or more solves of the PDE.

We will loosely follow [3,4], [2], [1] and include recent research literature.

We will rely on the material taught in Scientific Computing I (Wissenschaftliches Rechnen I). Knowledge of solving (elliptic and/or semilinear) PDEs will be advantageous. A basic understanding of minimization under constraints and variational calculus will be helpful.

The seminar presentations (50 minutes) should be self-consistent and understandable without requiring specialized prior knowledge. A four-page written summary in  $\text{\LaTeX}$  must be turned in by email or in print until July 31, 2014.

The seminar will take place on Mondays at 3:00pm s.t. in room 6.020, Wegelerstr. 6. We will have our first meeting and assign presentation topics on Monday, April 14.

## References

- [1] L. BIEGLER, G. BIROS, O. GHATTAS, Y. MARZOUK, M. HEINKENSCHLOSS, D. KEYES, B. MALLICK, , L. TENORIO, B. VAN BLOEMEN WAANDERS, AND K. WILLCOX, eds., *Large-scale Inverse Problems and Quantification of Uncertainty*, Wiley, 2011.
- [2] A. BORZÌ AND V. SCHULZ, *Computational Optimization of Systems Governed by Partial Differential Equations*, SIAM, 2012.
- [3] F. TRÖLTZSCH, *Optimale Steuerung partieller Differentialgleichungen*, Vieweg, Wiesbaden, Germany, 2005.
- [4] F. TRÖLTZSCH, *Optimal Control of Partial Differential Equations: Theory, Methods and Applications*, vol. 112 of Graduate Studies in Mathematics, American Mathematical Society, 2010.